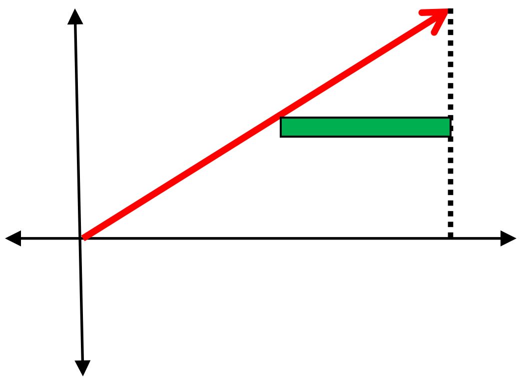
Unit 6: Volume of Solids
Revolution using Shells about the X and Y-axis

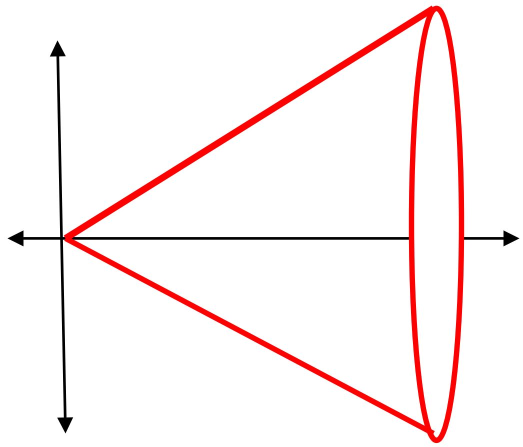
Name: ________________

SHELL METHOD

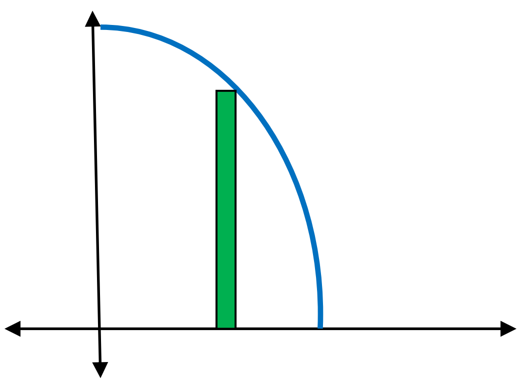
A shell is produced when you slice a rectangle in a region so that the length of the rectangle is **parallel** to the axis of rotation.

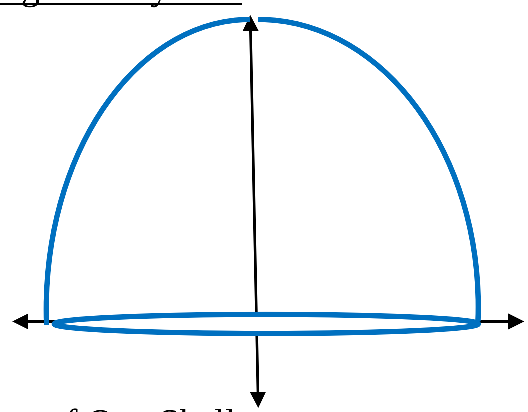
Rotating on the x-axis





Rotating on the y-axis





Volume of One Shell

$$V_{\text{Shell}} = lwh$$

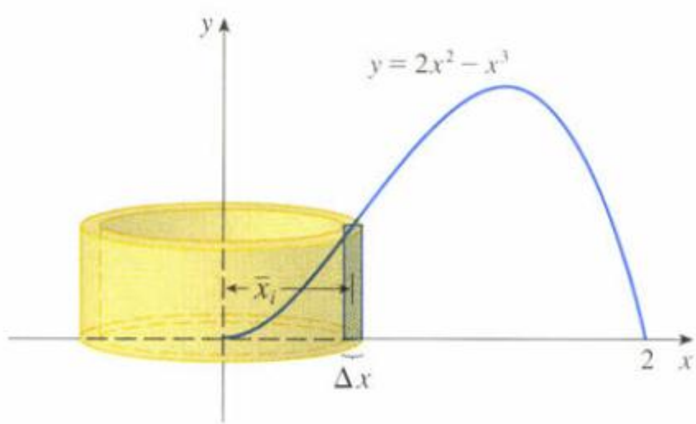


FIGURE 14 A cylindrical shell

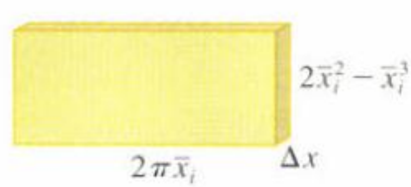


FIGURE 15 The flattened shell

Name: ________________

NOTE:
l = Circumference of shell $(2\pi r)$
w = dy or dx

$h = f(x), f(y)$ OR $f(x) - g(x)$, *or* $g(y) - f(y)$
r = x or y

(y-axis)

$$\boxed{\int_a^b 2\pi x f(x)dx}$$

(x-axis)

$$\boxed{\int_a^b 2\pi y f(y)dy}$$

Examples:

1. Use the shell method to find the volume of the solid generated by rotating the region between:
$f(x) = 4 - x^2$, y= 0, and x = 0 (about the y-axis)

2. Use the shell method to find the volume of the solid generated by rotating the region in between:
$f(x) = 5 - x$, y=0, and x= 0 (about the x-axis)

3. a. Use the shell method to find the volume of the solid generated by rotating the region in between:
$f(x) = x^2, g(x) = 3x$ (about the x-axis)

b. Now, validate your answer using the washer method.

4. Sketch the solid and setup the integral to find the volume generated by rotating the region between:
$y = \ln x,\ y = 0,\ x = e$ (about the line x=e)

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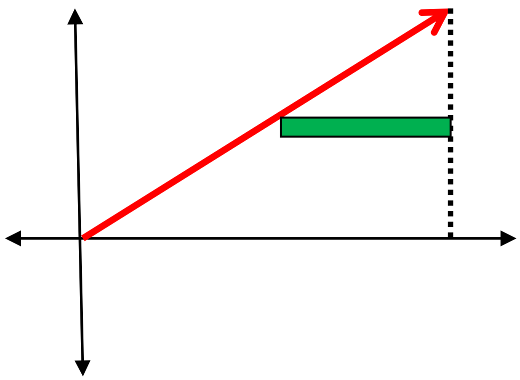
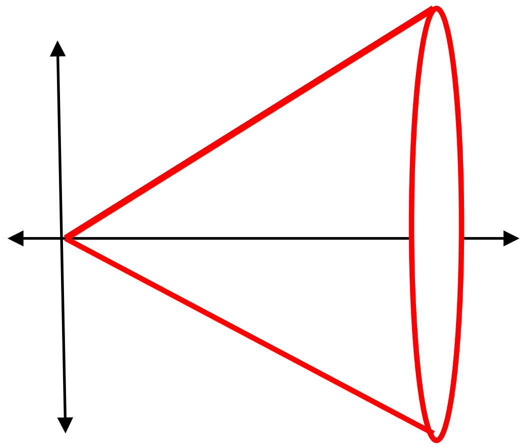
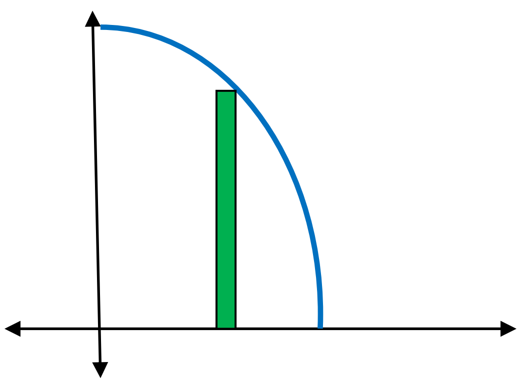
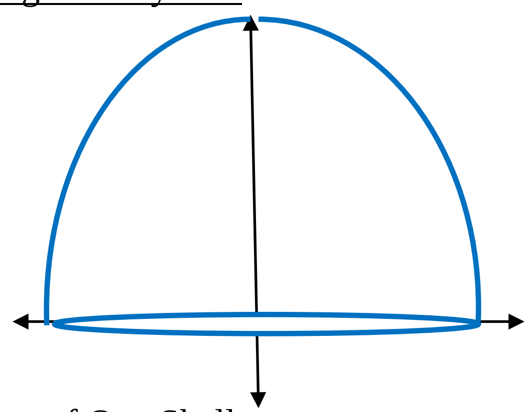
Rotating on the x-axis





Rotating on the y-axis





Volume of One Shell

$$V_{Shell} = lwh$$

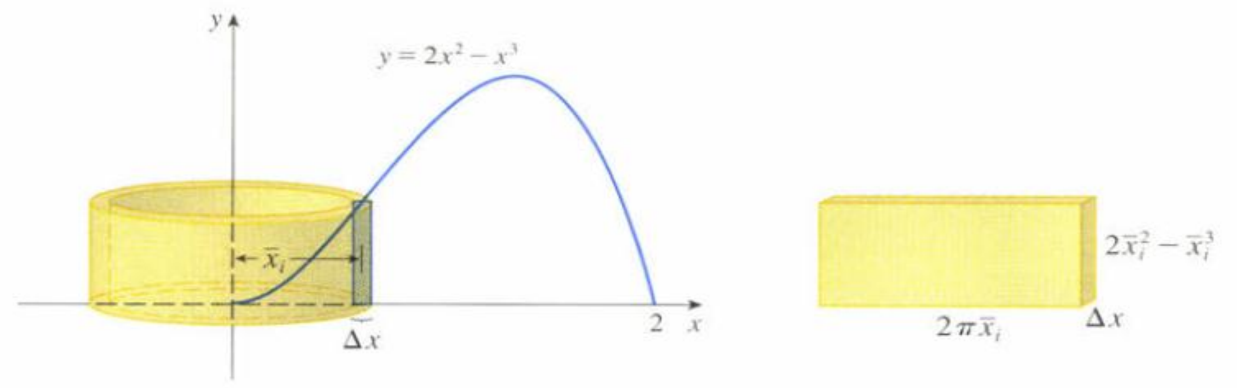


FIGURE 14 A cylindrical shell

FIGURE 15 The flattened shell

Name: ________________

NOTE:
l = Circumference of shell $(2\pi r)$
$w = dy$ or dx
$h = f(x), f(y)$ OR $f(x) - g(x)$, *or* $g(y) - f(y)$
$r = x$ or y

Examples:

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2. Use the shell method to find the volume of the solid generated by rotating the region in between:
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3. a. Use the shell method to find the volume of the solid generated by rotating the region in between:
   $f(x) = x^2, g(x) = 3x$ (about the x-axis)

   a. b. Now, validate your answer using the washer method.

4. Sketch the solid and setup the integral to find the volume generated by rotating the region between:
   $y = \ln x,\ y = 0,\ x = e$ (about the line x=e

Name: ________________

Solids of Revolution ***Shell Method***

1) Center of shell is the axis of rotation.
2) Radius is the distance from axis of rotation to the edge of the shell.
3) The height extends from the bottom to top (or left to right) of the region.
4) x represents the distance from the y-axis.
5) y represents the distance from the x-axis.

Washer / Disk Method vs. Shell Method

Axis of rotation:	Vertical	Horizontal
Washer/ Disk Method	**dy integration**	**dx integration**
Shell Method	**dx integration**	**dy integration**

Remember, you are finding the volume of the shell, not the volume of the cylinder.

Shell

Revolved around y-axis $\int_a^b 2\pi x f(x)dx$

Radius = x

Revolved around x-axis $\int_a^b 2\pi y f(y)dy$

Radius = y

*If not revolved around the x or y axis, must find the radius.

Revolved around y = -1

$$\int_a^b 2\pi(y+1)\,f(y)dy$$

Name: ________________

Unit 6 WS 5
AP Calculus AB

Sketch the region bounded by the graphs of the given equations and use the Shell Method to find the volume of the figure generated when that region is rotated about the indicated axis.

1. $y = \frac{4}{x},\ y = 0,\ x = 1,\ x = 4$ rotated about the *y*-axis.

2. $y = x^2 + 1,\ y = 0,\ x = 0,\ x = 2$ rotated about the *y*-axis.

3. $y = \sqrt{x},\ y = 0,\ x = 4$ rotated about (x=4).

4. $y = 4 - x^2,\ y = 0,\ x = -2$ rotated about (x= -2).

5. $y = x^2,\ y = 2x$ rotated about the *y*-axis.

6. $y = \frac{1}{4}x^3 + 2,\ y = 2 - x,\ x = 2$ rotated about the *y*-axis.

7. $y = \sqrt[3]{x},\ x = 0,\ y = 2$ rotated about the *x*-axis.

8. $y = -x + 3$, x = 0, y = 0 rotated about (y=3).

9. $y = \sqrt{x} + 1,\ y = x + 1,\ x = 0$ rotated about the x-axis